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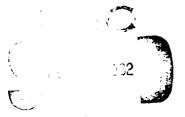




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Technical Report 81119

October 1981



LIFETIME PREDICTION FOR SATELLITES IN LOW-INCLINATION TRANSFER ORBITS

by

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LIFETIME PREDICTION FOR SATELLITES IN LOW-INCLINATION TRANSFER ORBITS

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D. G. King-Hele

SUMMARY

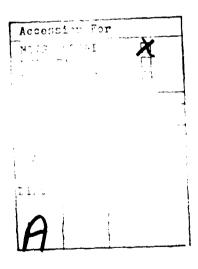
A geostationary satellite usually reaches its final circular orbit via a transfer orbit having apogee height near 36000 km and perigee height less than 600 km. The population of discarded rockets left in these transfer orbits is steadily increasing (there are now more than a hundred), and their likely lifetimes are usually assessed after lengthy numerical integrations to evaluate lunisolar perturbations. This paper gives a simple analytical method for predicting the lifetimes when the orbital inclination is between 20° and 30° , as often happens. Several approximations are made, but the errors should not exceed those from other sources.

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INTRODUCTION

Many geostationary satellites proceed to their final circular equatorial orbits at a height near 36000 km by way of transfer orbits having perigee heights near 200 km, apogee heights near 36000 km and inclinations of $20^{\circ}-30^{\circ}$. A discarded rocket usually remains in this transfer orbit, and its lifetime is often difficult to estimate because lunisolar perturbations substantially alter the perigee height as time passes. If the perigee height is pushed downwards initially, the rocket may decay within a year; if the perigee height is pushed upwards initially, the lifetime may be ten years or more.

To estimate the likely lifetimes of satellites in such orbits, it is usual to undertake extensive numerical integrations, which not only eat up computer time but may also suffer inaccuracy because the initial orbits are often in error by more than 10 km in perigee height. In this paper approximate analytical methods are devised for estimating lifetime, bearing in mind the likely 10 km error-level of the initial orbit.

Most of the objects to which the method applies are upper-stage rockets of satellites in series such as Intelsat, Fleetsatcom, Westar, Kiku, GOES and SBS. About forty such objects are currently in orbit, and eight were launched during the first nine months of 1981.

2 LIFETIMES IN THE ABSENCE OF LUNISOLAR PERTURBATIONS

A typical transfer orbit has a perigee height of about 200 km and apogee height near 35800 km. This means that the semi major axis a is near 24400 km, the orbital period T is near 0.438 day and the eccentricity e is near 0.73.

In the absence of lunisolar perturbations, the lifetime L of a satellite moving in an orbit of high eccentricity subject to air drag is given by equation (4.132) of Ref 2 as

$$L = \frac{(1-e)Tf(e)}{4\rho_{p}\delta} \left(\frac{1+e}{2\pi Hae}\right)^{\frac{1}{2}}, \qquad (1)$$

where f(e) is a function of e given in Table 4.2 of Ref 2 as 1.50 for e = 0.73; ρ_p and H are the density and density scale height at perigee; and δ is a drag parameter given by $\delta = \text{FSC}_D/\text{m}$. Here F is a factor allowing for atmospheric rotation, which has a value near 0.92 for low-inclination transfer orbits, C_D is the drag coefficient, which may be taken as 2.2 for heights near 200 km, and m/S is the mass/area ratio of the satellite.

Inserting the numerical values for the transfer orbit into equation (1), we obtain

$$L = \frac{23.6}{10^9 \rho_D \sqrt{H}} \left(\frac{m}{100S} \right) \quad \text{years}, \tag{2}$$

with m in kg, S in m², ρ_p in kg/m³ and H in km. A typical value of m/S is 100 kg/m² and we may take this as a standard value to give a standard lifetime L_0 ,

$$L_0 = \frac{23.6}{10^9 \rho_p \sqrt{H}}$$
 years. (3)

4

With values of ρ_p and H from the COSPAR International Reference Atmosphere 1972^3 for exospheric temperatures T_∞ of 700 K (low solar activity) and 1000 K (high solar activity), the variation of L_0 with perigee height is as given in Fig !. High solar activity is not usually maintained for more than about three years, and low solar activity does not last for more than about six years in each sunspot cycle. So if the calculated lifetime exceeds six years, the intermediate curve labelled 'medium solar activity' should be used.

Although 100 kg/m² is a useful standard value for m/S, the real value may be considerably different; the lifetime L_0 then needs to be multiplied by m/100S. In calculating m/S for a particular rocket, m should of course be taken as the mass after all the propellant has been consumed, and the mean cross-sectional area S is usually slightly less than the side area ℓ d of the rocket, if it is a cylinder of length ℓ and diameter d. If $\ell/d > 2$, the mean value of S may be taken² as ℓ d(0.82 + d/4 ℓ).

3 CHANGES IN PERIGEE HEIGHT DUE TO LUNISOLAR PERTURBATIONS

In the absence of air drag, the change in eccentricity due to lunisolar perturbations is given explicitly by G.E. Cook as the sum of 15 terms in equation (55) of Ref 4. If we insert numerical values for the orbital inclination of the satellite (25°) and the obliquity of the ecliptic (23.4°), and also take the lunar inclination as 23.4°, equation (55) may be written, on omitting terms less than 2% of the main term, as

$$\dot{e} = \frac{15K}{4n} e(1 - e^2)^{\frac{1}{2}} \left\{ 0.835 \sin 2(\Omega - \Omega_d - u_d + \omega) - 0.147 \sin(\Omega - \Omega_d + 2\omega) + 0.072 \sin 2(\Omega - \Omega_d + \omega) - 0.153 \sin(2u_d - \Omega + \Omega_d - 2\omega) + 0.068 \sin 2\omega \right\} , \qquad (4)$$

where K = 0.971 \deg^2/\deg^2 for the Sun, K = 2.132 \deg^2/\deg^2 for the Moon and n = 1/T = 2.28 rev/day = 821 deg/day. In equation (4), Ω and Ω_d are the right ascensions of the ascending node of the satellite and of the disturbing body respectively, ω is the satellite's argument of perigee, and u_d is the longitude of the disturbing body.

The changes in eccentricity due to lunisolar perturbations are always small, so that e can be taken constant on the right-hand side of equation (4). This assumption holds even in the presence of drag, because the lifetime will already be decided by the time when e decreases to 0.67, and since $0.497 \le e(1-e^2)^{\frac{1}{2}} \le 0.500$ for $0.67 \le e \le 0.73$, we may conveniently take $e(1-e^2)^{\frac{1}{2}} = 0.5$.

Consider first the solar perturbation. With the numerical values already quoted for K, n, etc, the term outside the curly brackets in equation (4) becomes 0.00222 deg/day = 0.0141 rad/yr. Also u_d is replaced by the solar longitude λ , and Ω_d = 0. Thus

$$\dot{e}_{solar} = 0.0141 \left\{ 0.835 \sin 2(\Omega + \omega - \lambda) - 0.147 \sin(\Omega + 2\omega) + 0.072 \sin 2(\Omega + \omega) - 0.153 \sin(2\lambda - \Omega - 2\omega) + 0.068 \sin 2\omega \right\} .$$
 (5)

For a transfer orbit at an inclination near 25° , $\mathring{\Omega} \simeq -0.4$ deg/day $\simeq -2.5$ rad/yr, and $\mathring{\omega} \simeq 0.7$ deg/day $\simeq 4.4$ rad/yr. Also $\mathring{\lambda} = 6.3$ rad/yr. So, preparing for integration by inserting the rates of change divided by their numerical values, we have

$$\dot{e}_{solar} = -0.00141 \Big[0.949 \Big\{ 2(\hat{n} + \dot{\omega} - \dot{\lambda}) \Big\} \sin 2(\Omega + \omega - \lambda) + 0.233(\hat{n} + 2\dot{\omega}) \sin(\Omega + 2\omega) \\ -0.189 \Big\{ 2(\hat{n} + \dot{\omega}) \Big\} \sin 2(\Omega + \omega) + 0.243(2\dot{\lambda} - \dot{n} - 2\dot{\omega}) \sin(2\lambda - \Omega - 2\omega) \\ -0.077(2\dot{\omega}) \sin 2\omega \Big] .$$
 (6)

Integrating, we find the variation of e with time t due to solar perturbations as

$$\left(e - e_0\right)_{\text{solar}} = 0.00141 \left[0.949 \cos 2(\Omega + \omega - \lambda) + 0.233 \cos(\Omega + 2\omega) - 0.189 \cos 2(\Omega + \omega) + 0.243 \cos(\Omega + 2\omega - 2\lambda) - 0.077 \cos 2\omega\right]_0^t ,$$
 (7)

where suffix 0 denotes initial values.

Now consider the lunar perturbations. On inserting numerical values, the term outside the square brackets in (4) becomes 0.0310 rad/yr. Since \dot{u}_d = 84 rad/yr for the Moon, the first term in equation (4) has a frequency of 2 weeks and its amplitude is less than 10% of the main solar variation in (7). So it may be ignored in our long-term analysis. The fourth term in (4) can also be ignored, being of short period and much smaller amplitude. On integrating the three remaining terms in (4), we find, in analogy with (7),

$$\left(e - e_0\right)_{1\text{unar}} = 0.00310 \left[0.233 \cos(\Omega - \Omega_d + 2\omega) - 0.189 \cos 2(\Omega - \Omega_d + \omega) - 0.077 \cos 2\omega\right]_0^t .$$
 (8)

As $\Omega_{\rm d}$ has a mean value of zero over 18.4 years, and $|\Omega_{\rm d}|$ never exceeds 13°, we may conveniently take $\Omega_{\rm d}$ = 0 in (8). Adding (7) and (8) gives the total lunisolar perturbation as

$$e - e_0 = \left[0.00134 \cos 2(\Omega + \omega - \lambda) + 0.00105 \cos(\Omega - 2\omega) - 0.00085 \cos 2(\Omega + \omega) + 0.00034 \cos(\Omega + 2\omega - 2\lambda) - 0.00035 \cos 2\omega\right]_0^t . \tag{9}$$

On multiplying by a (= 24400 km) to convert from eccentricity to perigee height y_p , and rounding off to the nearest km, we have:

$$y_p - y_{p0} = \left[33 \cos 2(\Omega + \omega - \lambda) + 26 \cos(\Omega + 2\omega) - 21 \cos 2(\Omega + \omega) + 8 \cos(\Omega + 2\omega - 2\lambda) - 9 \cos 2\omega \right]_{+}^{0} km.$$
 (10)

From equation (10), the mean perigee height \bar{y}_p over several cycles is given by

$$\overline{y}_{p} = y_{p0} + 33 \cos 2(\Omega_{0} + \omega_{0} - \lambda_{0}) + 26 \cos(\Omega_{0} + 2\omega_{0}) - 21 \cos 2(\Omega_{0} + \omega_{0}) + 8 \cos(\Omega_{0} + 2\omega_{0} - 2\lambda_{0}) - 9 \cos 2\omega_{0} \quad km.$$
 (11)

As a first approximation we may ignore the last two terms, of amplitude 8 and 9 km, and also the effects of odd zonal harmonics⁵, which produce an oscillation with an amplitude of about 3 km. Then

$$\bar{y}_{p} = y_{p0} + \Delta y_{p1} + \Delta y_{p2} + \Delta y_{p3}$$
 say, (12)

where $\Delta y_{pl} = 33 \cos 2(\Omega_0 + \omega_0 - \lambda_0)$, $\Delta y_{p2} = 26 \cos(\Omega_0 + 2\omega_0)$ and $\Delta y_{p3} = -21 \cos 2(\Omega_0 + \omega_0)$ are given in Fig 2. Thus, if the initial orbital elements of the satellite are known, we can calculate $y_{p0} = a_0(1 - e_0) - 6378$ km, and then calculate \bar{y}_p from (12) or, more accurately, from (11).

4 WEIGHTED MEAN PERIGEE HEIGHT FOR USE IN LIFETIME PREDICTION

The arithmetic mean perigee height \overline{y}_p is not the correct mean value to use in calculating lifetime, because the variation of density with height is strongly exponential, and not linear. For estimating lifetime we need to use the height y^* at which the perigee density ρ_p has its mean value ρ^* , such that

$$\rho^* = \frac{1}{P} \int_0^P \rho_p dt \tag{13}$$

over a complete period P of the oscillation in perigee height. If we now further approximate the variation (10) as a simple sinusoid of amplitude $\sqrt{33^2 + 26^2 + 21^2} = 47$ km and frequency f, say, we have $y_p = y_m + 47(1-\cos ft)$, where y_m is the minimum perigee height. Since the perigee density is greatest at $y_p = y_m$, its variation with height should be expressed in a form which is correct at and near $y_p = y_m$, that is

$$\rho_{\rm p} = \rho_{\rm m} \exp \left\{ - \left(y_{\rm p} - y_{\rm m} \right) / H_{\rm m} \right\} = \rho_{\rm m} \exp \left\{ - 47 (1 - \cos ft) / H_{\rm m} \right\},$$
 (14)

where H_m is the density scale height at height y_m . On writing ft = τ and integrating from 0 to 2π , equations (13) and (14) give:

$$\rho^* = \frac{\rho_{\rm m}}{2\pi} \exp(-47/H_{\rm m}) \int_{0}^{2\pi} \exp(47 \cos \tau/H_{\rm m}) d\tau . \qquad (15)$$

Using the first of equations (14) to express ρ^* in terms of ρ_m , we can rewrite (15) as

$$\exp\left\{-(y^* - y_m)/H_m\right\} = \left\{\exp(-47/H_m)\right\} I_0(47/H_m), \qquad (16)$$

where $I_0(z)$ is the Bessel function of the first kind and imaginary argument of degree zero and argument z. Solving (16) for y* gives

$$y^* = y_m + 47 - H_m \ln \left\{ I_0(47/H_m) \right\} . \tag{17}$$

Since $y_m + 47 = \overline{y}_p$, we may rewrite (17) as

$$y^* = \bar{y}_p - \Delta y_{p4} , \qquad (18)$$

where $\Delta y_{p4} = H_m \ln \left\{ I_0(47/H_m) \right\}$ and is plotted against \bar{y}_p in Fig 3.

Thus the procedure for lifetime prediction is to evaluate \tilde{y}_p from equation (11) by calculation, or (less accurately) from (12) and Fig 2, and then to find y* from (18) by reading off Δy_{p4} from Fig 3. The lifetime of the satellite can then be obtained from Fig 1, taking the perigee height in Fig 1 to be y*.

A further simplification of equation (11) is possible if ω_0 is near 0 or 180°, as often happens because the apogee of the transfer orbit is usually required to be near the equator. If $\omega_0 = 0$ or 180°, equation (11) reduces to

$$\bar{y}_p = y_{p0} + 33 \cos 2(x_0 - \lambda_0) + 26 \cos x_0 - 21 \cos 2x_0 + 8 \cos(x_0 - 2\lambda_0) - 9 \text{ km}.$$
 (19)

Although the approximate method is intended primarily for satellites with perigee height near 200 km, there are some objects (such as Comstar rockets) in transfer orbits with a much higher perigee, near 600 km. The method can also be applied to these objects, since the changes in the numerical factors are not significant. So, to complete the picture, Fig 4 gives the variation of lifetime with perigee height for transfer orbits with perigee heights between 300 and 700 km. The curved labelled $T_{\infty} = 900 \text{ K}$ would be applicable if the average future solar activity remains at the levels prevalent during the 20th century, but the prediction of solar activity for up to 100000 years ahead is a matter of some uncertainty.

5 EFFECT OF CHANGING THE INCLINATION

The analysis has been pursued on the assumption that the orbital inclination i is 25°, but there will be a range of inclination over which the results are likely to be within the required limits of accuracy. The validity of the method at other inclinations can be assessed by looking at the three main components of the variation in perigee height.

The variation of perigee height y_p given by subtracting equation (11) from equation (10) is

$$y_p - \bar{y}_p = -33 \cos 2(\Omega + \omega - \lambda) - 26 \cos(\Omega + 2\omega)$$

+ 21 \cos 2(\Omega + \omega) - 8 \cos(\Omega + 2\omega - 2\lambda) + 9 \cos 2\omega \text{km}. (20)

The first term on the right-hand side, of amplitude 33 km, has a period of about 9 months; the second term, of amplitude 26 km, has a period of about 12 months; and the third term, of amplitude 21 km, has a period of about 20 months. The two smaller terms have periods of 12 and 9 months respectively. If air drag seriously affects the orbit, these regular sinusoidal oscillations will of course gradually be modified, but if the lifetime is 5 years or more, the first few cycles will be little affected by drag and equation (20) should be realistic, for $i = 25^{\circ}$.

A change in inclination from 25° affects both the periods and amplitudes of the oscillations. Increasing the inclination reduces the period and amplitude of the first term, and increases those of the second and third terms. If the inclination is increased to 30°, the three amplitudes (33, 26 and 21 km) become 29, 35 and 25 km respectively, while the periods change from about 9, 12 and 20 months to about 8, 14 and 25 months. If the inclination is reduced to 20°, the amplitudes become 37, 19 and 18 km, and the periods become 9, 11 and 17 months. Thus, none of the amplitudes changes by more than 9 km, and the root sum of the squares of the amplitudes does not change by more than 5 km. So it is apparent that equation (20) can be used for any inclination i between 20° and 30°: the errors will be somewhat greater when i departs from 25°, but should still not exceed 10 km (sd), and can be reduced by using revised amplitudes in accord with those quoted above. Also, of course, 1 and 4 change at different rates if i departs from 25°.

If the inclination is not too far outside the range $20^{\circ}-30^{\circ}$, it is still possible to use an equation of the form (20) but with the amplitudes adjusted to suit the inclination. If we rewrite the three main terms in equation (20) as:

$$y_{p} - \overline{y}_{p} = -A_{1} \cos 2(\Omega + \omega - \lambda) - A_{2} \cos(\Omega + 2\omega) + A_{3} \cos 2(\Omega + \omega) , \qquad (21)$$

the amplitudes A_1 , A_2 and A_3 vary with inclination in the manner shown in Fig 5. The lower limit of inclination in Fig 5 is extended to 10° so as to cover the Ariane third stage 1981-57B, and 30° is a convenient upper limit because there are as yet no rockets in transfer orbits at inclinations between 30° and 40° .

Fig 5 can of course also be used in the procedure for lifetime prediction. If the inclination is outside the range $20^{\circ}-30^{\circ}$, the mean perigee height should be calculated using equation (11) with values of A_1 , A_2 and A_3 from Fig 5 to replace the values 33, 26 and 21 km given in equation (11). (The same procedure could also be used for inclinations between 20° and 30° ; but, as already discussed earlier in this section, the improvement in accuracy would only be marginal.) Fig 5 also shows that $\sqrt{A_1^2 + A_2^2 + A_3^2}$ remains between 45 and 52 km for inclinations between 10° and 30° , so there is no need to modify the analysis in section 4. When i changes to 10° the amplitudes of the fourth and fifth terms in equation (20) change from 8 and 9 km to 5 and 1 km respectively, so they can be ignored.

The method developed here fails when the orbit is near-resonant, as happens at $i \simeq 46^\circ$ when $\dot{\Omega} + \dot{\omega} \simeq 0$ and A_3 becomes very large; or at $i \simeq 56^\circ$ when $\dot{\Omega} + 2\dot{\omega} \simeq 0$ and A_2 becomes very large. Another resonance occurs at $i = 63.4^\circ$ when $\dot{\omega}$ becomes

very small and the amplitude of the term in $\cos 2\omega$ in equation (20) therefore becomes very large: this is the situation for the Molniya satellites⁶. Because of these three resonances, the approach adopted here only succeeds for inclinations up to about 35°.

testing the theory against numerical integration

To conduct a test, we need a satellite with a lifetime of at least 5 years and inclination near 25°. The example chosen is 1978-71C, the third stage of ESA-Geos 2, which was launched in July 1978 and was left in a transfer orbit of inclination 25.4°, eccentricity 0.729 and semi major axis 24300 km: these values are close to those assumed in developing the approximate method.

Fig 6 shows, as an unbroken line, the variation of y_p in 1979-80 as given by numerical integration with the program $PROD^7$, with 5-day integration intervals. The small lunar perturbations, of amplitude about 4 km and period 2 weeks, have been smoothed out. (For pictorial examples of these oscillations, see Fig 4 of Ref 8, or Ref 9.) The broken line in Fig 6 shows the variation of y_p given by

$$y_p = 240 - 33 \cos 2(\Omega + \omega - \lambda) - 26 \cos(\Omega + 2\omega) + 21 \cos 2(\Omega + \omega)$$
 km, (22)

with the values of $\,\Omega\,$ and $\,\omega\,$ taken from PROD (though a linear variation of $\,\Omega\,$ and $\,\omega\,$ would have been adequate).

The forms of the two curves in Fig 6 are very similar. The difference between them has a mean numerical value of 8 km and a maximum of 18 km. This is as would be expected when terms of amplitude 8 and 9 km have been ignored.

7 ACCURACY

There are two main orbital errors in the use of the approximate method. First, there is the error due to the approximation, which will be largest when the initial point happens to be on a relatively ill-fitting region, such as August 1979 or February 1980 in Fig 6. The error from this source is probably about 10 km (sd), and can be reduced if necessary by adding the two extra terms on the right-hard side of equation (11). Second, and probably worse, there are the errors of the initial orbit. The source of the initial orbit is sometimes a transfer orbit of the joined rocket-and-satellite: this is accurately known to the launch authority, but it may be unreliable if the kick at separation diverts the discarded rocket into a slightly different orbit. Alternatively, the orbit may be derived from subsequent radar observations; but unless the satellite is observed near perigee, it is extremely difficult with orbits of $e \approx 0.7$ to determine perigee height accurately from normal radar observations. Bias errors of 20 km or more can easily occur.

The errors due to both these sources can be checked and reduced by recalculating the lifetime with an 'initial' orbit a few months later, unless the object has been lost - which is often a signal of a rapid decay rate.

Other errors arise in estimating the mass/area ratio of the satellite and in predicting the average solar activity during the lifetime. These would both be expected to give errors of 10-20% in lifetime, unless the size, shape and weight of the rocket are

unknown, when the errors would be larger. It would then be appropriate to use the observed rate of decay during the early days in orbit to estimate the mass/area ratio.

8 AN_ALTERNATIVE_PROCEDURE

If a satellite in transfer orbit is successfully tracked for several cycles of the perigee height oscillation, is for about 5 years, the most accurate method of calculating lifetime is to use the mean observed orbital decay rate over the 5 years in conjunction with the normal graphical-analytical methods for predicting lifetime 10. This procedure avoids the effects of errors in the initial crbit and mass/area ratio, though the errors due to uncertainty in future solar activity are increased. The procedure also suffers from two limitations which do not affect the method developed in this paper: the satellite may be lost, or may decay, before the procedure can legitimately be applied.

9 CONCLUSION

An approximate analytical method has been developed for predicting the lifetimes of objects in transfer orbits with perigee height between 120 and 300 km, apogee height near 36000 km and inclination between 20° and 30° , with the aim of avoiding the need for numerical integration with each new launch.

There are four sources of error:

- (I) Estimating the mass/area ratio of the satellite;
- (2) Predicting the average solar activity during the lifetime;
- (3) Frrors in perigee height of the initial orbit used;
- (4) Errors in the estimated mean perigee height, caused by the approximations.

Sources (1) and (2) would both be expected to give errors of 10-20% in lifetime. Sources (3) and (4) would both be expected to give errors of about 10 km (sd) in perigee height, equivalent to errors by a factor of two in lifetime for low perigee (130 km height) or 20% error for high perigee (250 km). The errors (1) to (3) will remain, so there seems little point in trying to avoid error (4) by lengthy numerical integrations. The approximate method is to be preferred for lifetime predictions of newly-launched objects in low-inclination transfer orbits.

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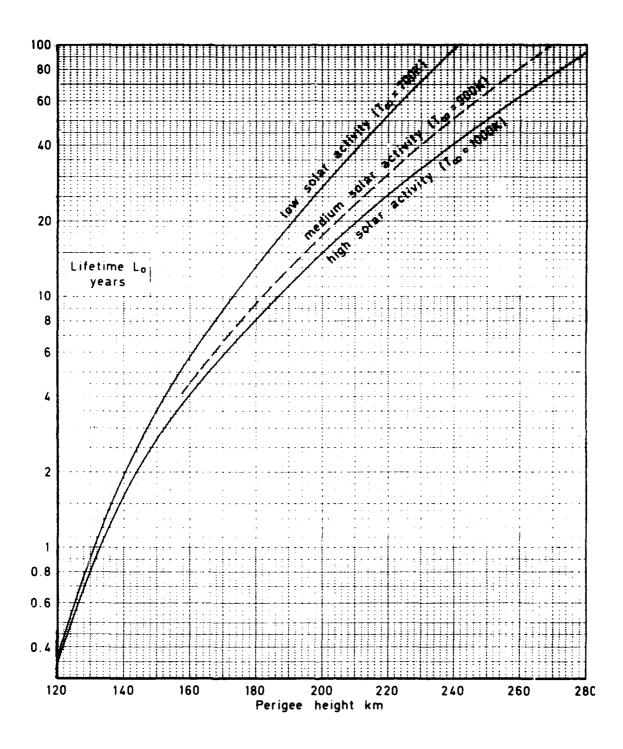


Fig 1 Lifetime L_0 of satellite of mass/area m/S \approx 100 kg/m² in transfer orbit of eccentricity 0.73. If m/S \neq 100 , multiply L_0 by m/100S. Lunisolar perturbations ignored

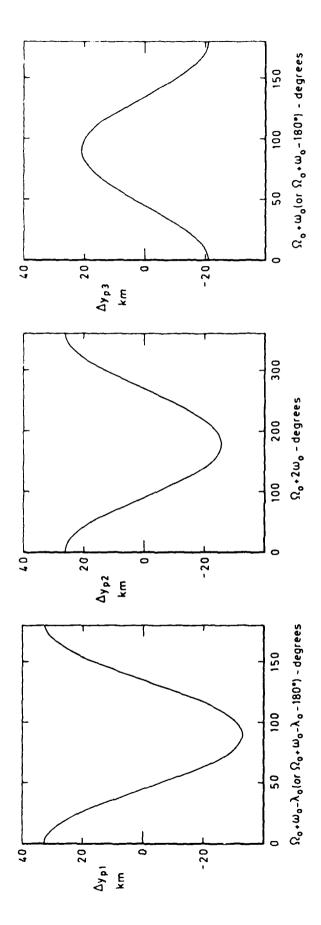


Fig 2 Charts for finding Δy_{p1} , Δy_{p2} and Δy_{p3}

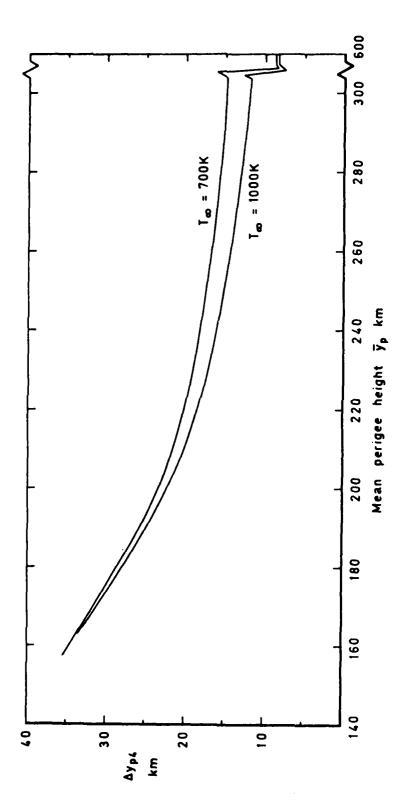


Fig 3 Chart for ∆yp4

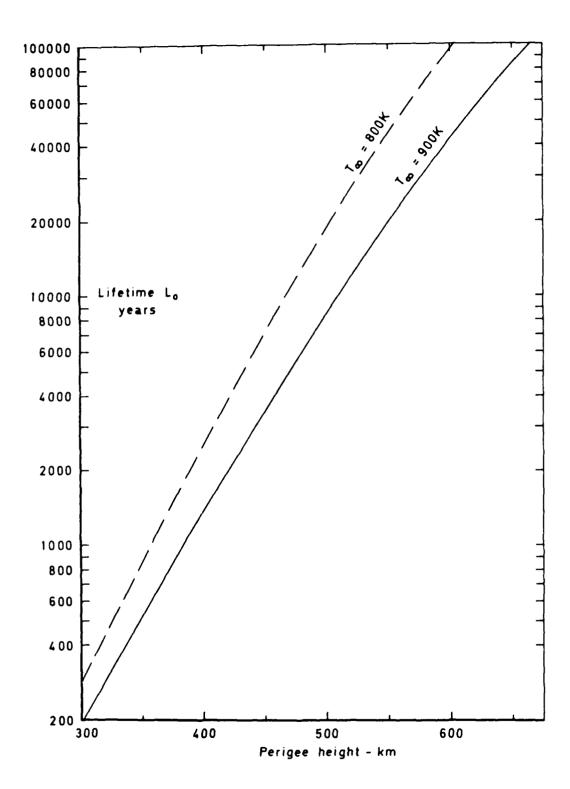


Fig 4 Lifetime L₀ of satellite of mass/area 100 kg/m² in transfer orbit with perigee height between 300 and 700 km, if the average future exospheric temperature is 800K or 900K

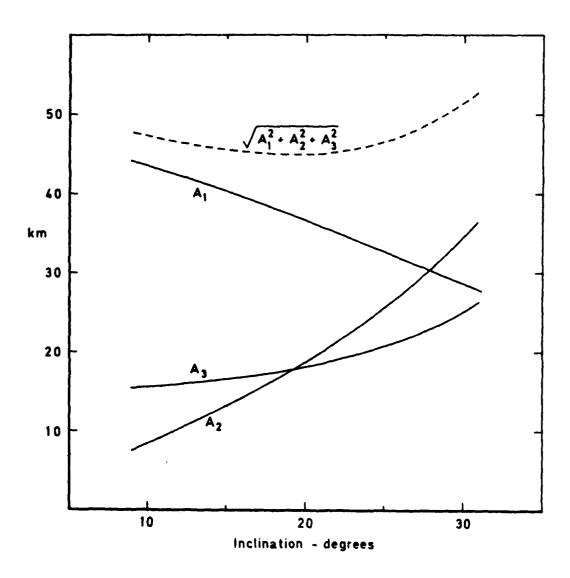


Fig 5 Variation of $\,{\rm A}_1^{}$, $\,{\rm A}_2^{}$ and $\,{\rm A}_3^{}$ with inclination

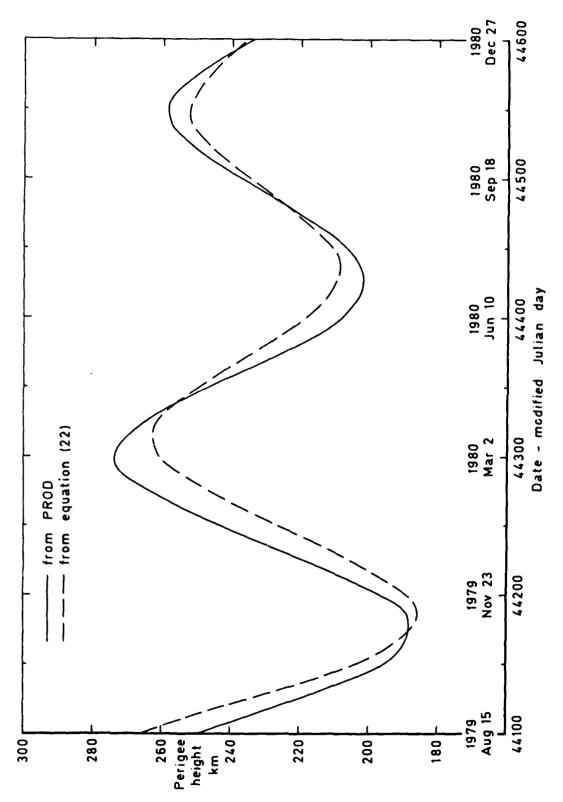


Fig 6 The variation of perigee height for 1978-71C as given by numerical integration (PROD) and by the approximate equation (22)

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A geostationary satellite usually reaches its final circular orbit via a transfer orbit having apogee height near 36000 km and perigee height less than 600 km. The population of discarded rockets left in these transfer orbits is steadily increasing (there are now more than a hundred), and their likely lifetimes are usually assessed after lengthy numerical integrations to evaluate lunisolar perturbations. This paper gives a simple analytical method for predicting the lifetimes when the orbital inclination is between 20° and 30°, as often happens. Several approximations are made, but the errors should not exceed those from other sources.

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